Lecture 10

Other Models of Turing Machines

COSE215: Theory of Computation

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Fall 2023

Contents

- **Other models of Turing machines**

Other models

- **A standard Turing machine is not the only possible one**
- **Turing machines with more complex storage**
  - TMs with a Stay-option
  - Multitape TMs
  - Multitrack TMs
  - Nondeterministic TMs
  - But these are equivalent with standard Turing machines!

Turing machines with a Stay-option

- **A Turing machine can have a Stay-option**
  - Possible movements of the head in a standard Turing machine
    - Left and Right
  - Possible movements of the head in a Turing machine with a Stay-option
    - Left, Right, and Stay

Turing machines with a Stay-option

- **Formal definition**
  - A Turing machine (TM) is a 7-tuple: $\boldsymbol{M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)}$
    - ❖ Q is a finite set of **internal states**
    - ❖ Σ is a finite set of **symbols**
      - $\Sigma \subseteq \Gamma - \{\square\}$
    - ❖ Γ is a finite set of symbols called **tape alphabets**
    - ❖ δ is a set of **transition functions**
      - $\boldsymbol{\delta: Q \times \Gamma \to Q \times \Gamma \times \{L, R, S\}}$
    - ❖ $q_0 \in Q$ is **the initial state**
    - ❖ $\square \in \Gamma$ is a special symbol called the **blank**
    - ❖ $F \subseteq Q$ is a set of **final states**

Turing machines with a Stay-option

- **TMs with a Stay-option vs Standard TM**
  - Is TM with a Stay-option more powerful?

Turing machines with a Stay-option

- **TMs with a Stay-option vs Standard TM**
  - Is TM with a Stay-option more powerful?
    - **No!**
  - We can build an equivalent standard TM for the given TM with a Stay-option

Turing machines with a Stay-option

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  - Is TM with a Stay-option more powerful?
    - **No!**
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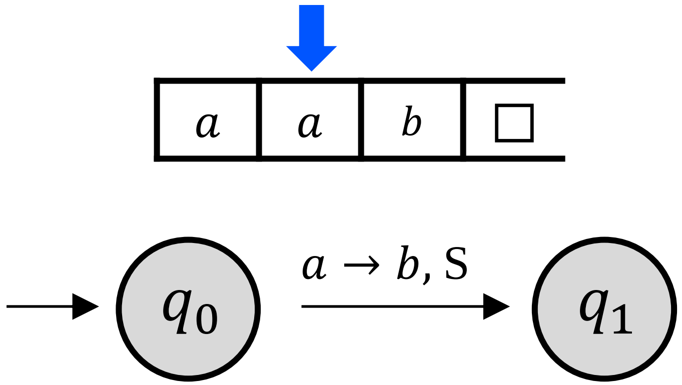

Turing machines with a Stay-option

- **TMs with a Stay-option vs Standard TM**
  - Is TM with a Stay-option more powerful?
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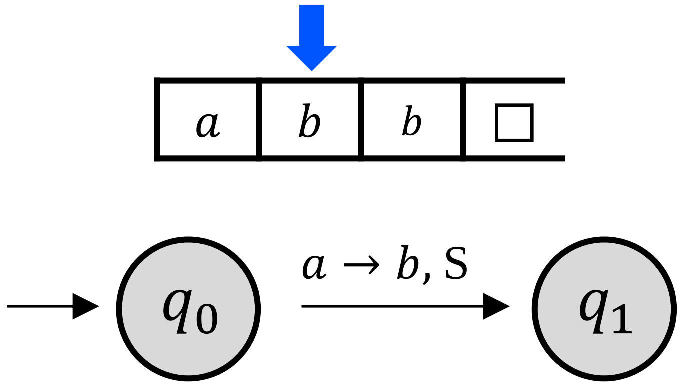

Turing machines with a Stay-option

- **TMs with a Stay-option vs Standard TM**
  - Is TM with a Stay-option more powerful?
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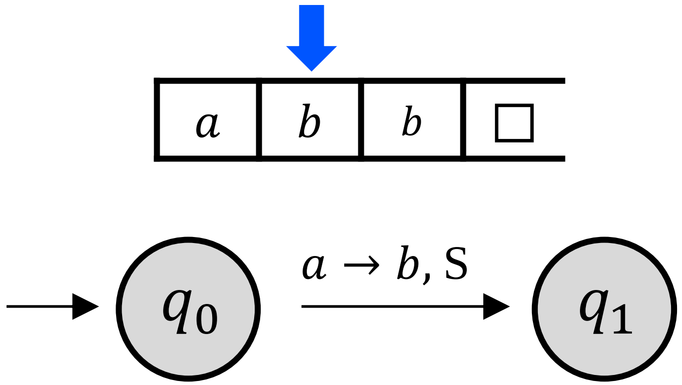

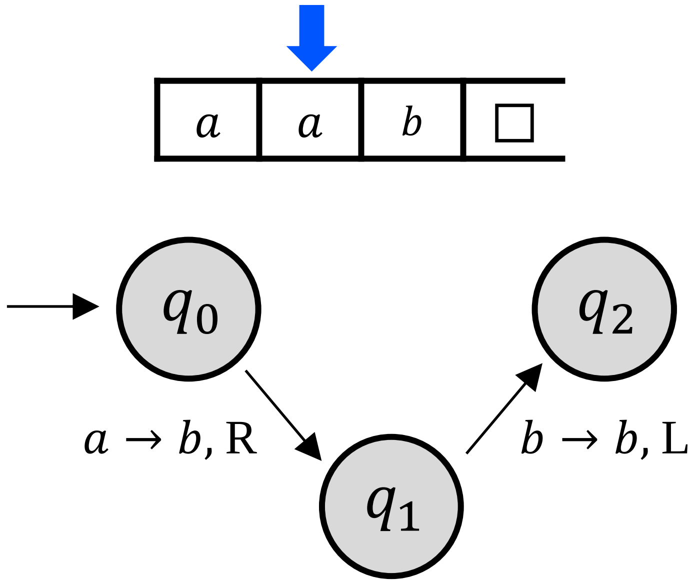

Turing machines with a Stay-option

- **TMs with a Stay-option vs Standard TM**
  - Is TM with a Stay-option more powerful?
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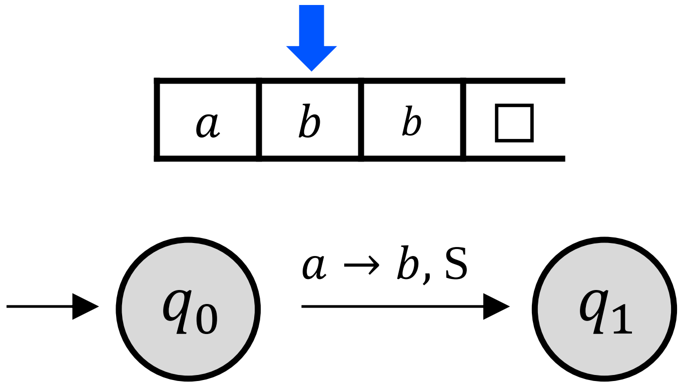
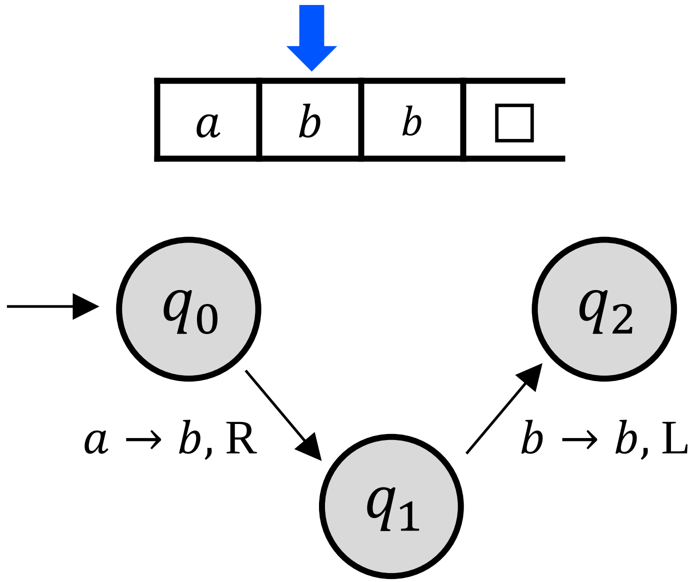
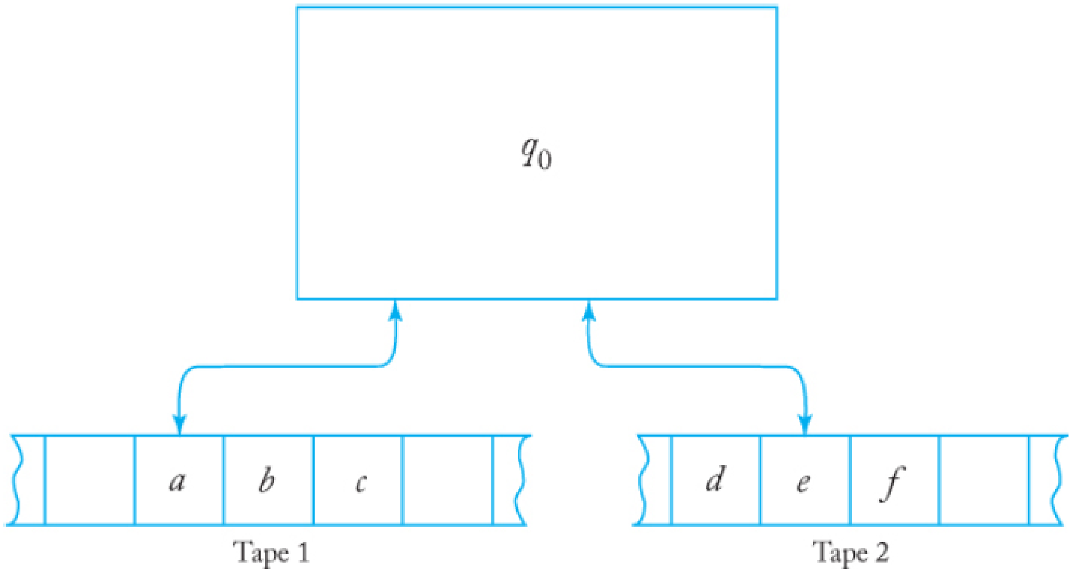




Multitape Turing machines

- **A Turing machine with several tapes**
  - Each with its own independently controlled read-write head



Multitape Turing machines

- **Multitape Turing machine: Formal definition**
  - A Turing machine (TM) is a 7-tuple: $\boldsymbol{M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)}$
    - ❖ Q is a finite set of **internal states**
    - ❖ Σ is a finite set of **symbols**
      - $\Sigma \subseteq \Gamma - \{\square\}$
    - ❖ Γ is a finite set of symbols called **tape alphabets**
    - ❖ δ is a set of **transition functions**
      - $\boldsymbol{\delta: Q \times \Gamma^{n} \rightarrow Q \times \Gamma^{n} \times \{L, R\}^{n}}$
    - ❖ $q_0 \in Q$ is **the initial state**
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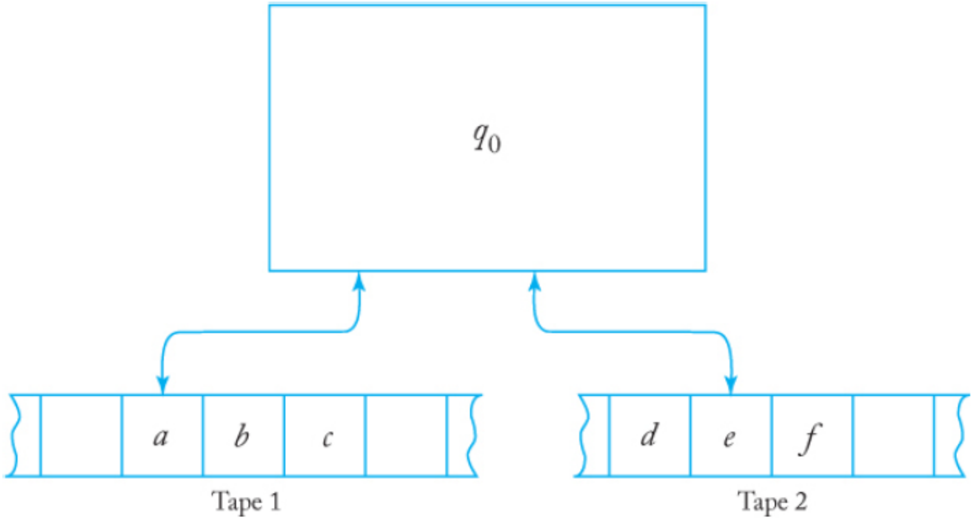
Multitape Turing machines

- **Multitape Turing machine: Example**
  - $\delta(q_0, a, e) = (q_1, x, y, L, R)$



Multitape Turing machines

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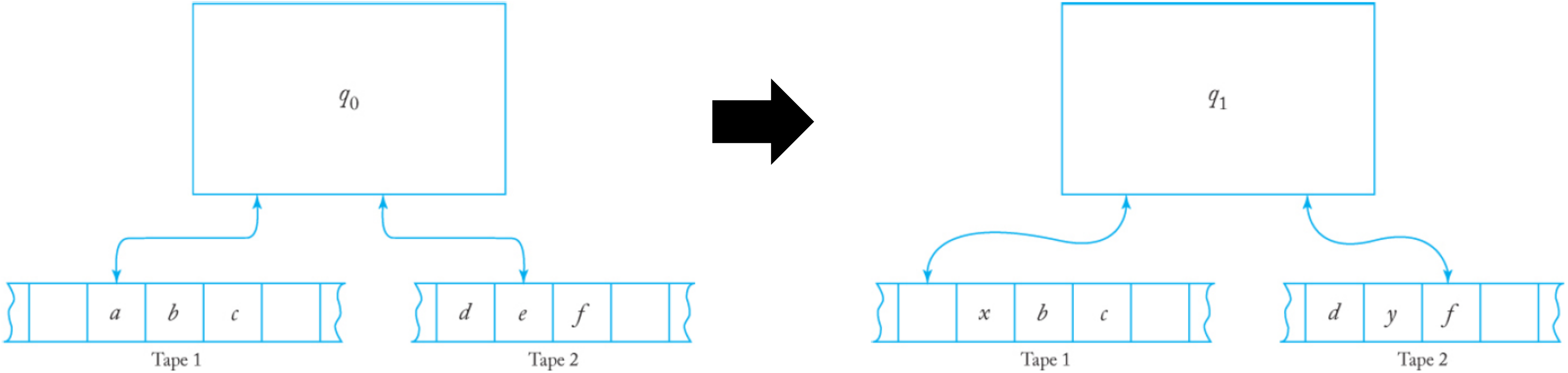


Multitape Turing machines

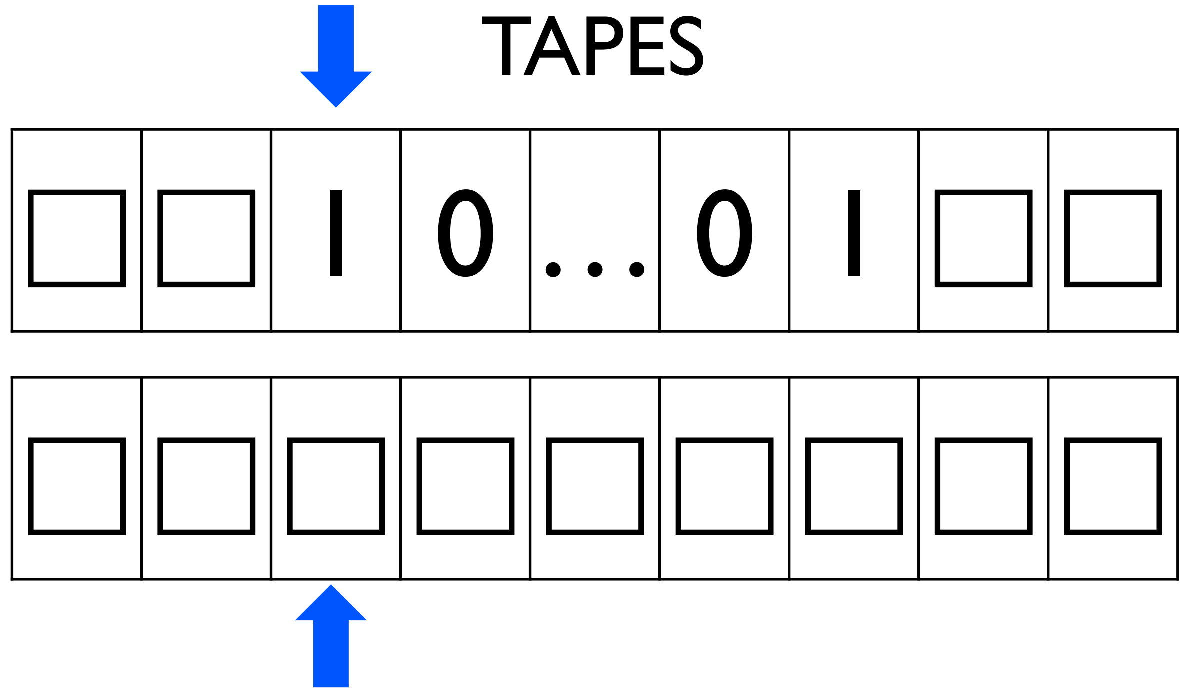
- **Multitape Turing machine: Example**
  - $L = \{\, w \mid w \text{ is a palindrome } (w = w^R) \text{ and } w \in \{0,1\}^* \}$
  - In a single tape TM,

TAPE

□	□	1	0	…	0	1	□	□

Multitape Turing machines

- **Multitape Turing machine: Example**
  - $L = \{ w \mid w \text{ is a palindrome } (w = w^R) \text{ and } w \in \{0,1\}^* \}$
  - In a multitape TM,



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- **Multitape Turing machine: Example**
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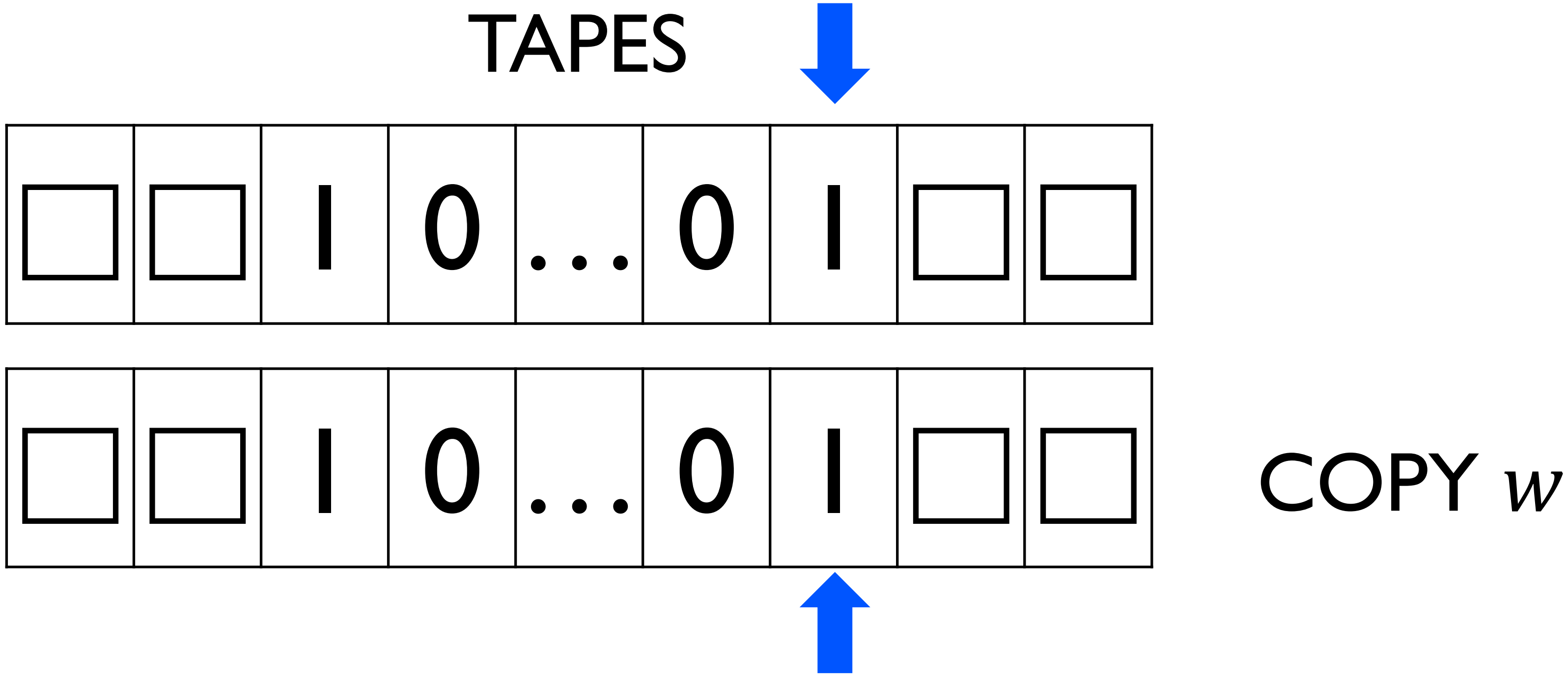

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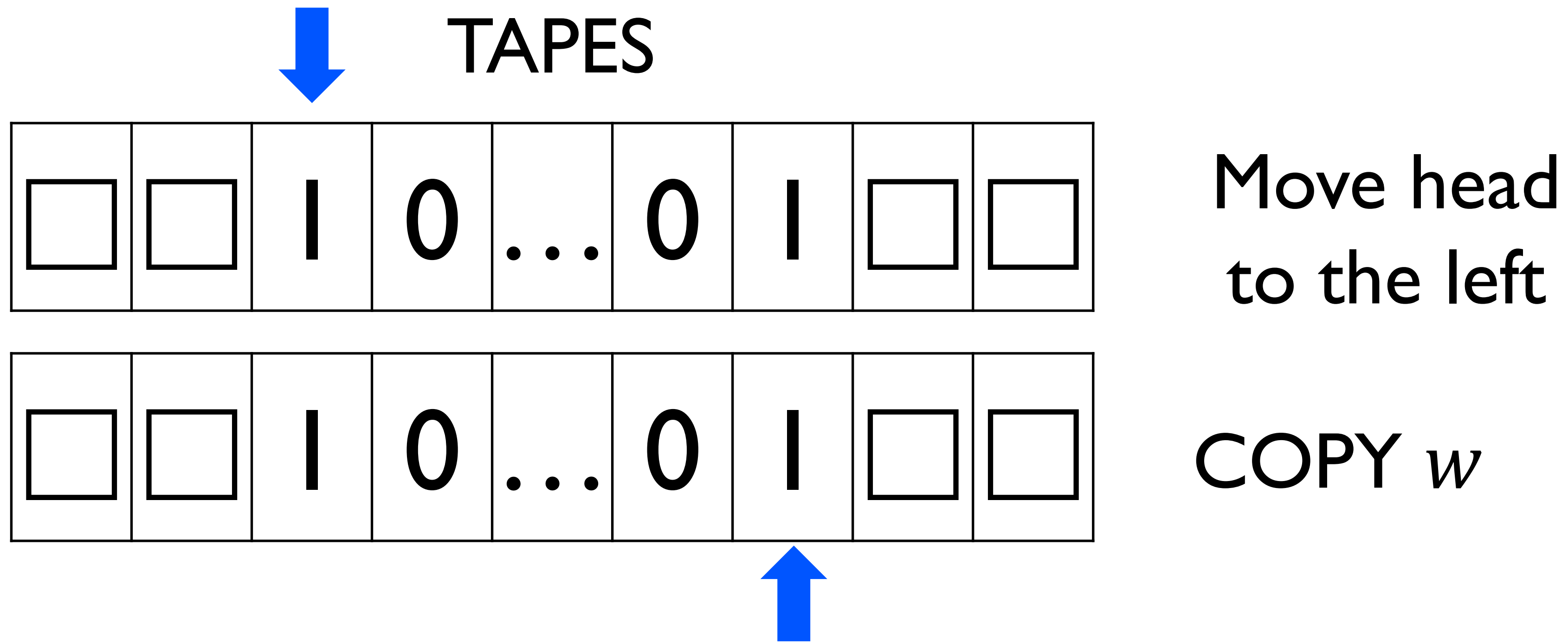

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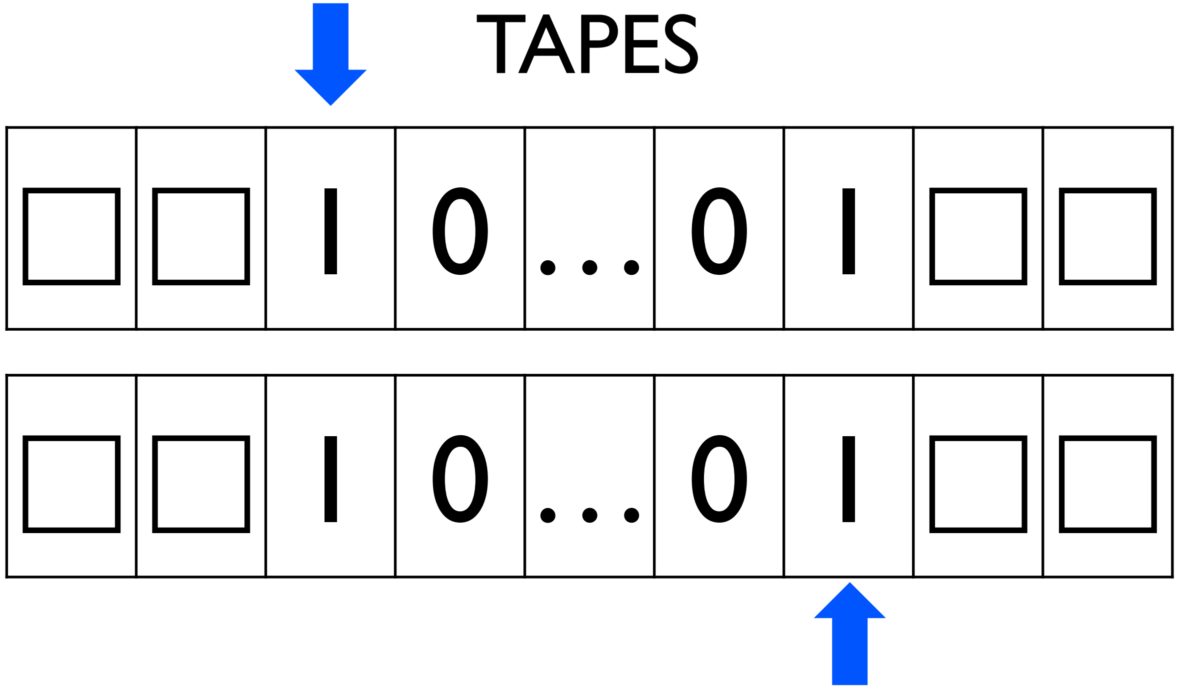

Compare
Two strings!

Multitape Turing machines

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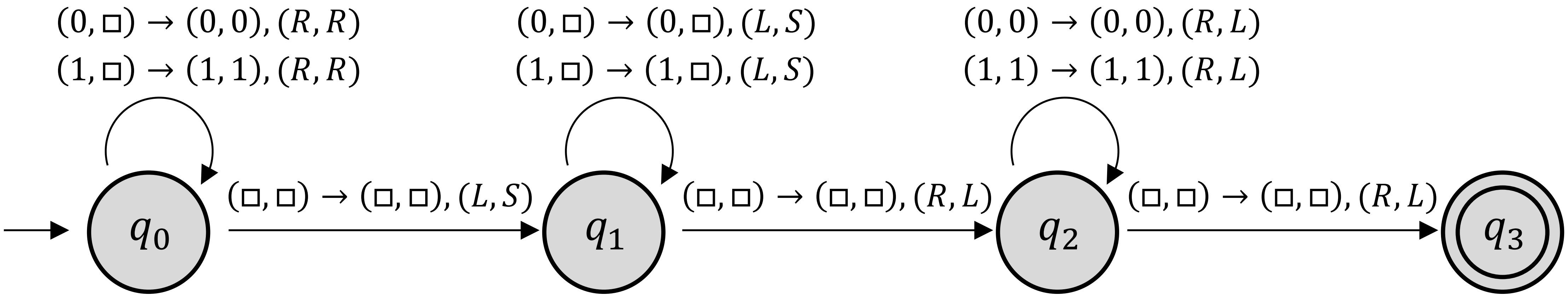


Multitape Turing machines

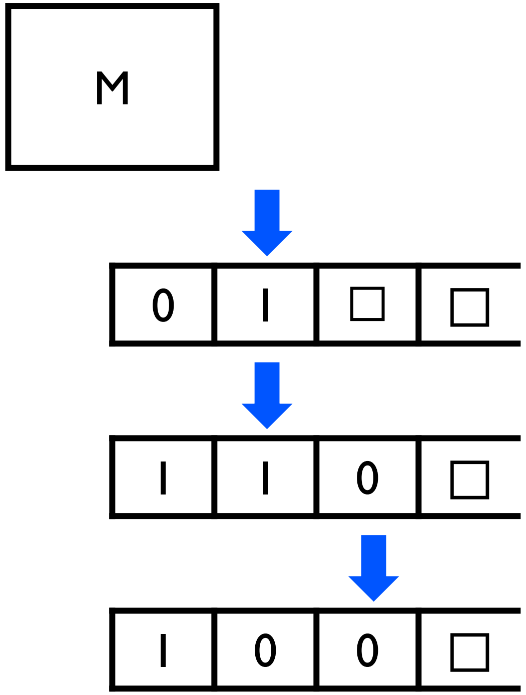
- **Multitape TM vs Standard TM**
  - Is multitape TM more powerful?

Multitape Turing machines

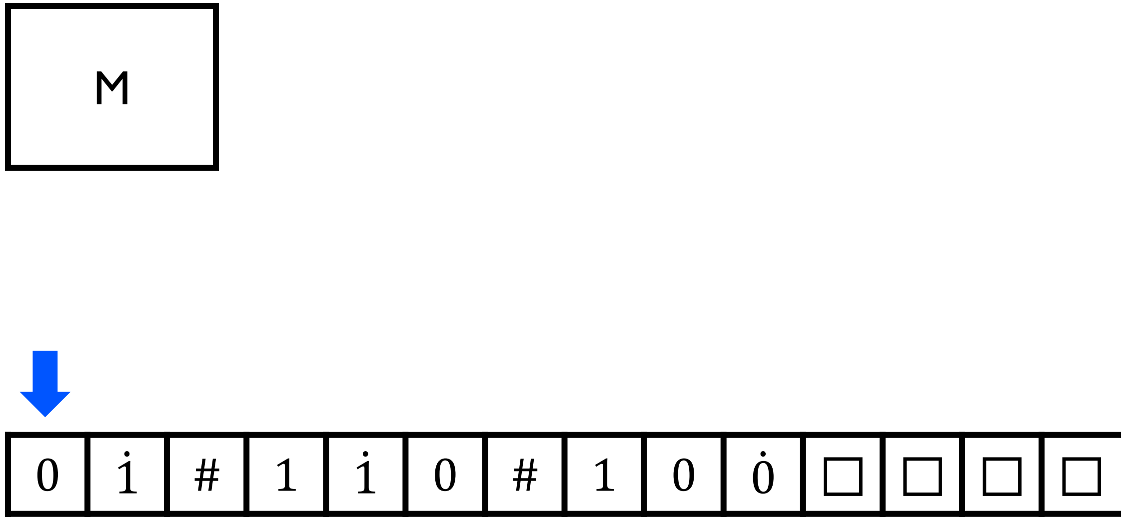
- **Multitape TM vs Standard TM**
  - Is multitape TM more powerful?
    - **No!**
  - We can build an equivalent standard TM for the given multitape TM

Multitape Turing machines

- **Multitape TM vs Standard TM**
  - One idea: using “#” and dotted symbols



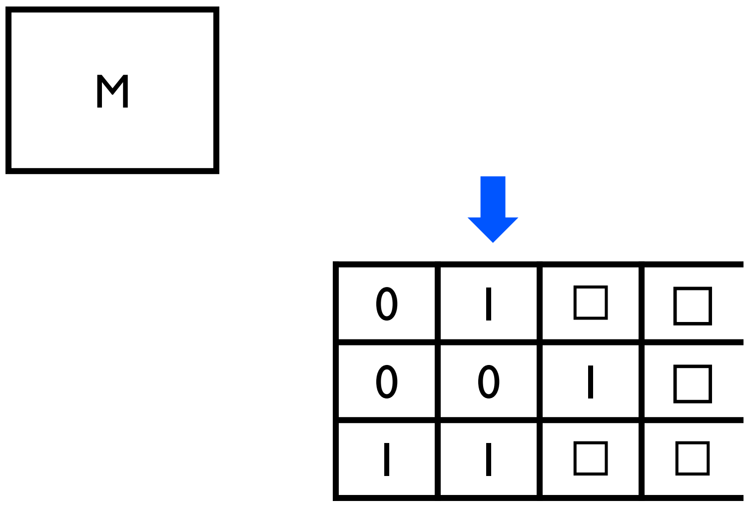
Multitape Turing Machine



Standard Turing Machine

Multitrack Turing machines

- **A Turing machine with multitrack tapes**



Multitrack Turing machines

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Multitrack Turing machines

- **Multitrack Turing machine: Example**
  - Multitrack TM for $L = \{a^n b^n c^n \mid n \geq 0\}$

Multitrack Turing machines

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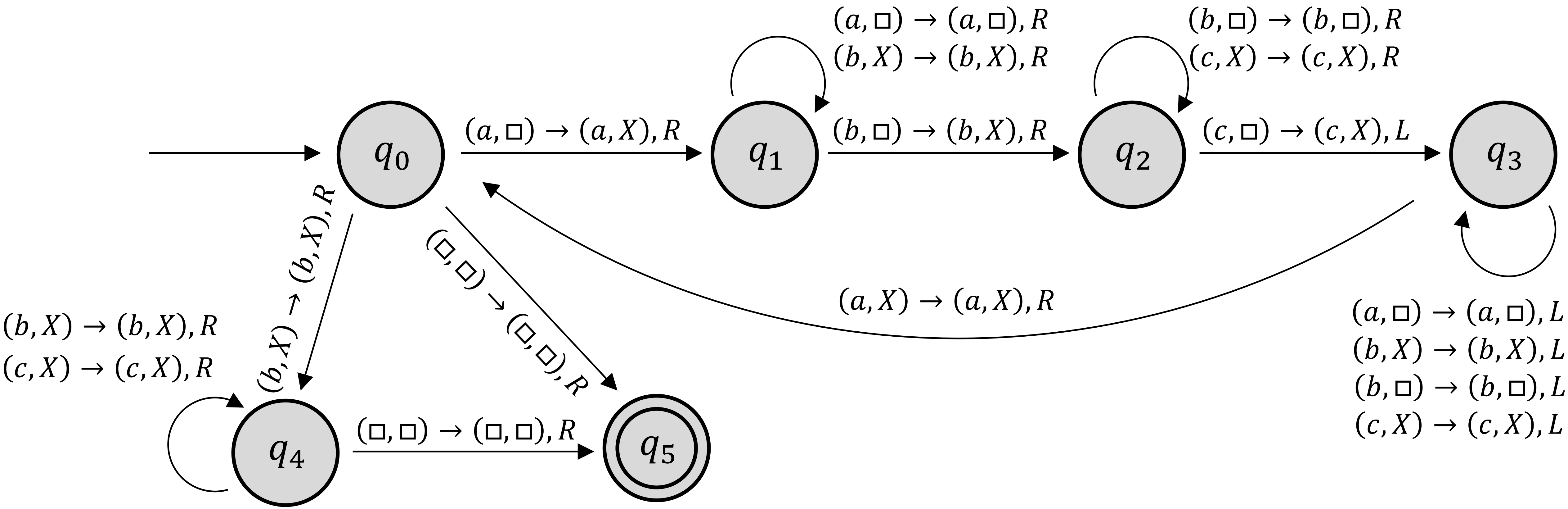
  - Multitrack TM for $L = \{a^n b^n c^n \mid n \geq 0\}$



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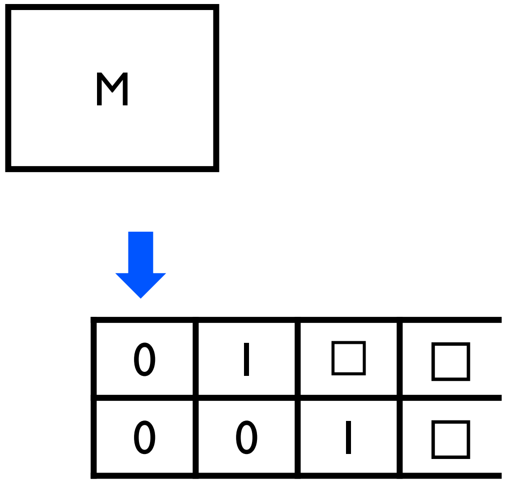
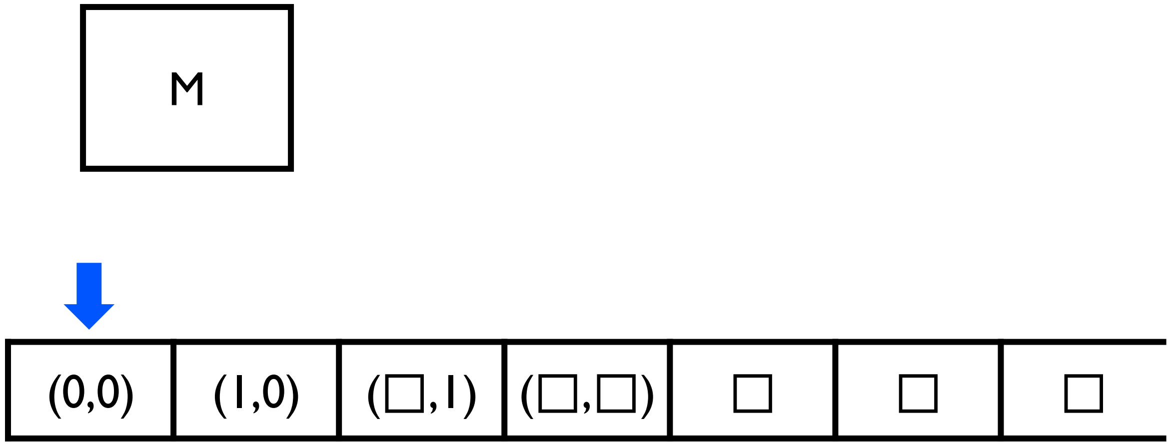
Multitrack Turing machines

- **Multitrack TM vs Standard TM**



Multitrack Turing Machine



Standard Turing Machine

Nondeterministic Turing machines

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    - ❖ Γ is a finite set of symbols called **tape alphabets**
    - ❖ δ is a set of **transition functions**
      - $\boldsymbol{\delta: Q \times \Gamma \to 2^{Q \times \Gamma \times \{L,R\}}}$
    - ❖ $q_0 \in Q$ is **the initial state**
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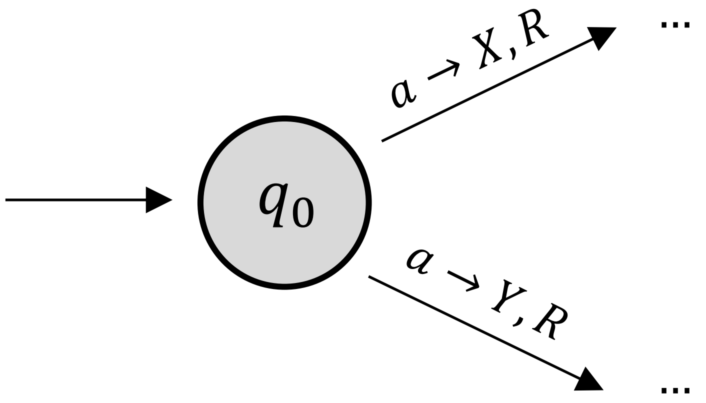
Nondeterministic Turing machines

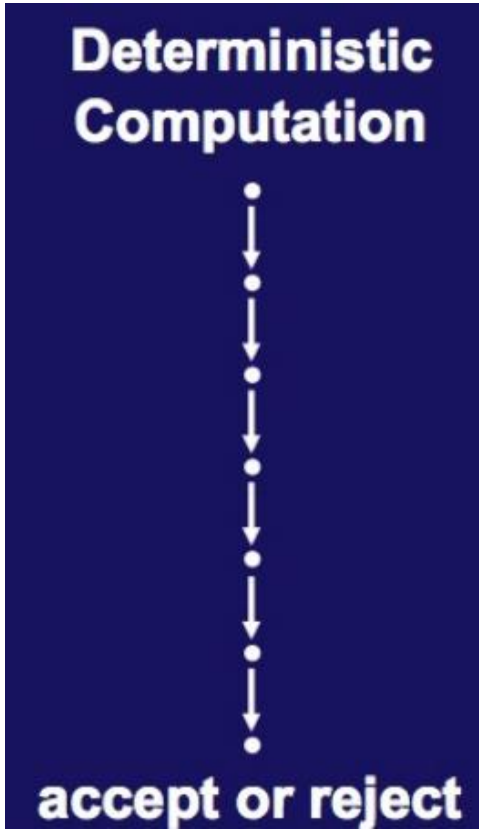
- **Nondeterministic Turing machine**

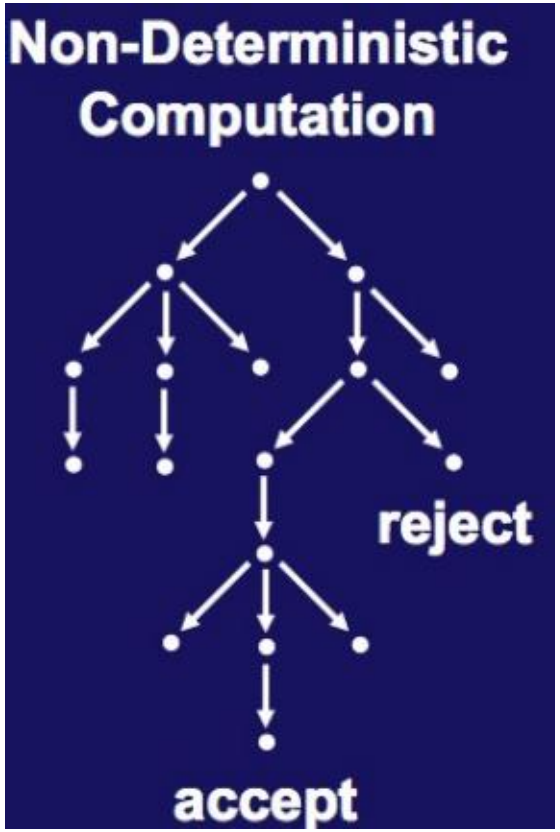


Nondeterministic Turing machines

- **It is not clear what role NTM plays in computing functions..**
  - NTM are usually viewed as accepters
- **We can build an equivalent standard TM for the given NTM**

Nondeterministic Turing machines

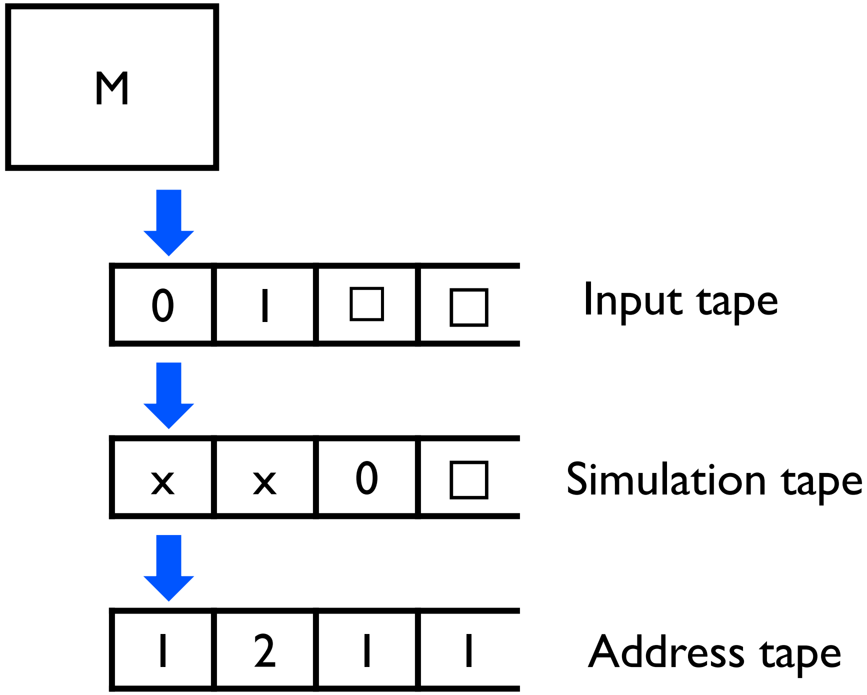




<https://www.andrew.cmu.edu/user/ko/pdfs/lecture-13.pdf>

Nondeterministic Turing machines

- **Leverage three-tape Turing machine**



Nondeterministic Turing machines

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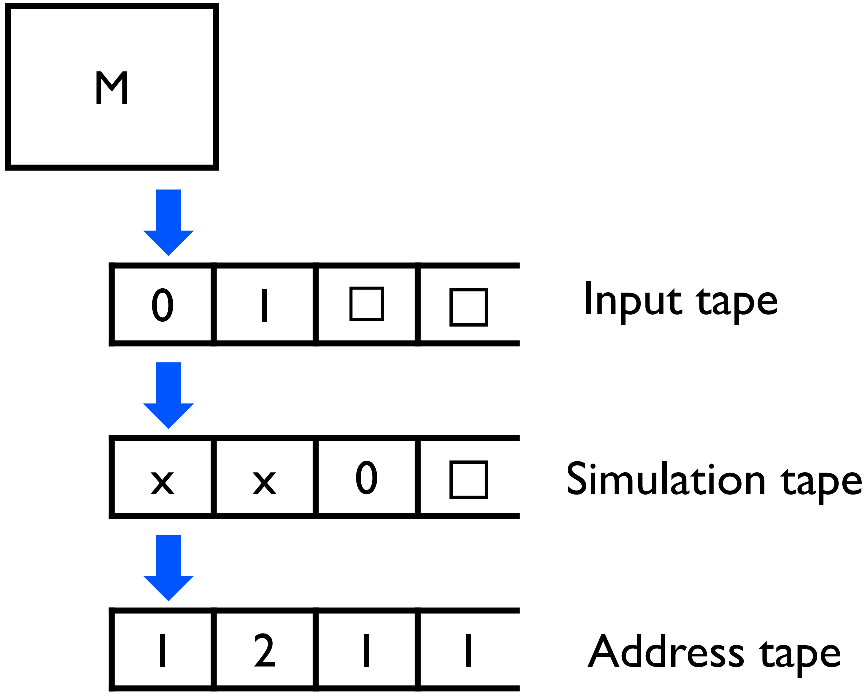

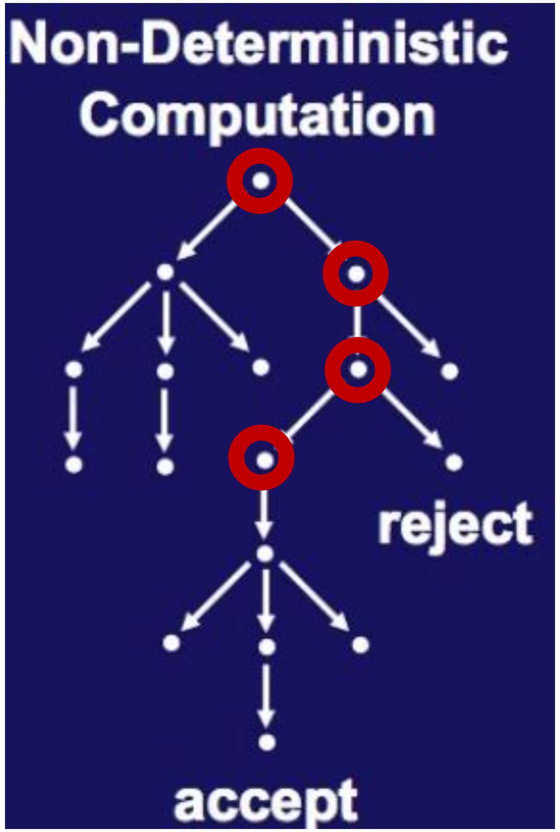

More extensions of TMs

- **There are more extensions**
  - TMs with k-head tape
  - TMs with semi-infinite tape
  - …
- **But all of them are equivalent to standard TMs**
  - A standard TM is the most powerful model of computation!

Next Lecture

- **Decidability**
- **Chomsky Hierarchy**