Lecture 4

Properties of Regular Languages

COSE215: Theory of Computation

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Practice for Lecture 3

- **Find a DFA for the given regular expression, and then extract regular grammar**
  - $(1(0+1)+00^*1)0^*1^*$

Contents

- **Closure properties of regular languages**

Closure properties of regular languages

- **What happens performing operations on regular languages**
  - E.g., Given two regular languages L_1 and L_2: Is their union also regular?

Closure properties of regular languages

- **What happens performing operations on regular languages**
  - E.g., Given two regular languages L_1 and L_2: Is their union also regular?
    - YES
    - Regular languages are **closed** under **union**

Closure properties of regular languages

- **What happens performing operations on regular languages**
  - If L_1 and L_2 are regular languages, then so are
    - $L_1 \cup L_2$ (UNION)
    - $L_1 \cap L_2$ (INTERSECTION)
    - $L_1 \cdot L_2$ (CONCATENATION)
    - $L_1 - L_2$ (DIFFERENCE)
    - $\overline{L_1}$ (COMPLEMENTATION)
    - L_1^* (STAR)

Closure properties of regular languages

- **Remember last week's lecture**

Regular Expressions and regular languages

- **If r is a regular expression, then $L(r)$ is a regular language**
  - A language is regular if it is accepted by a DFA
  - We can construct an NFA that accepts $L(r)$ for any regular expression r
    - NFA => DFA (equivalence)

Closure properties of regular languages

- **If L_1 and L_2 are regular languages, then so is $L_1 \cup L_2$**
  1. Let r_1 and r_2 be the regular expressions such that $L(r_1) = L_1$ and $L(r_2) = L_2$
  2. Note that $L(r_1) \cup L(r_2) = L(r_1 + r_2)$
  3. Here, $r_1 + r_2$ is the regular expression
  4. Therefore, $L_1 \cup L_2$ is a regular language

Closure properties of regular languages

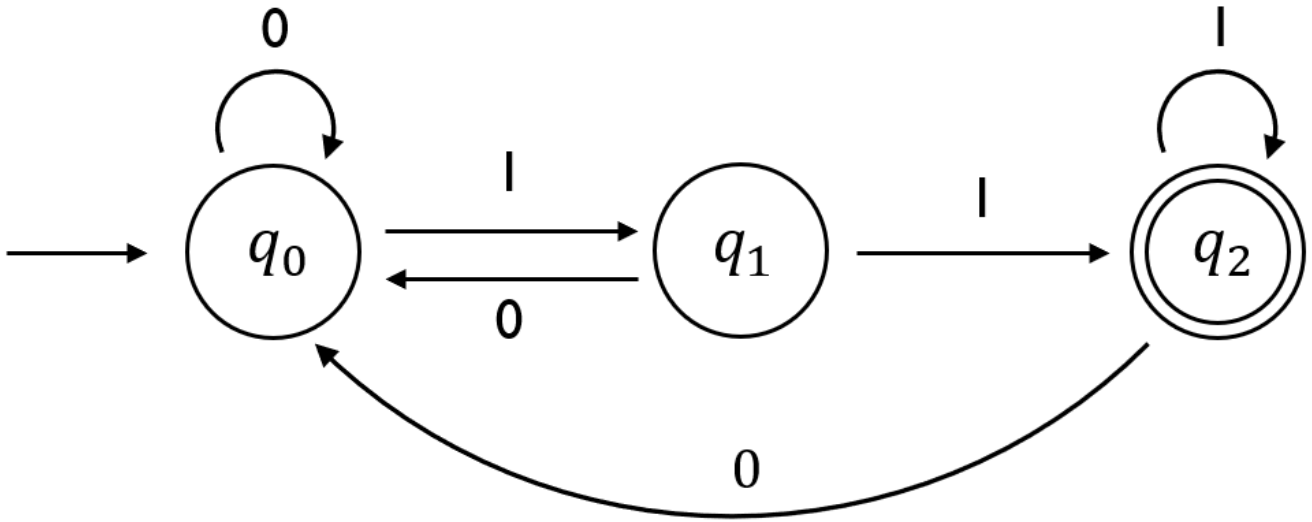
- **If L_1 and L_2 are regular languages, then so is $L_1 \cdot L_2$**
  1. Let r_1 and r_2 be the regular expressions such that $L(r_1) = L_1$ and $L(r_2) = L_2$
  2. Note that $L(r_1) \cdot L(r_2) = L(r_1 \cdot r_2)$
  3. Here, $r_1 \cdot r_2$ is the regular expression
  4. Therefore, $L_1 \cdot L_2$ is a regular language

Closure properties of regular languages

- **If L_1 is a regular language then so is L_1^***
  1. Let r_1 be the regular expression such that $L(r_1) = L_1$
  2. Note that $\left(L(r_1)\right)^* = L(r_1^*)$
  3. Here, r_1^* is the regular expression
  4. Therefore, L_1^* is a regular language

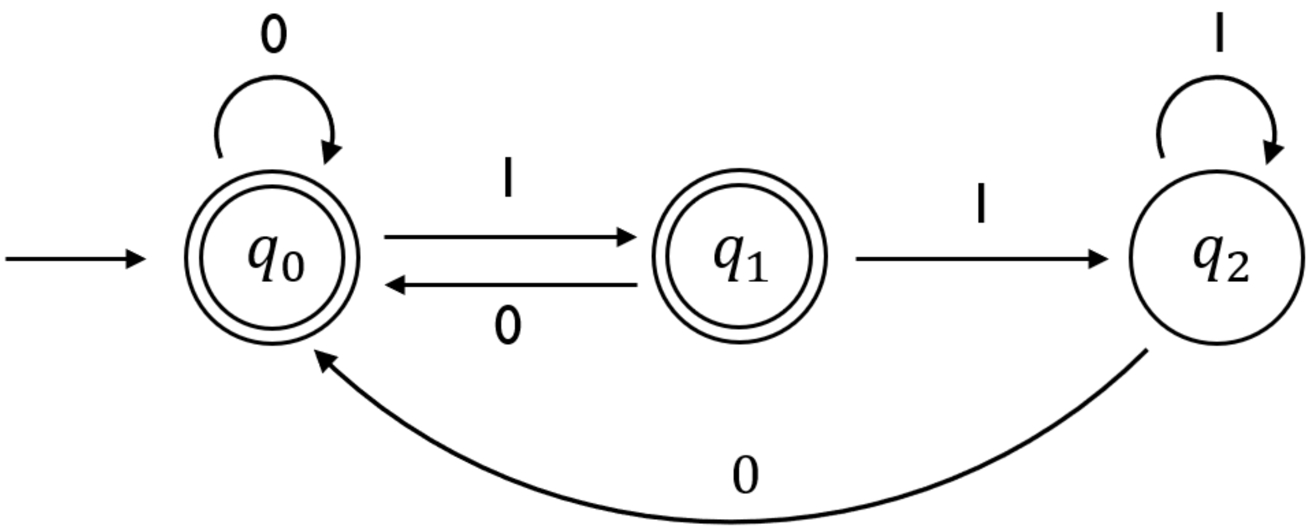
Closure properties of regular languages

- **If L_1 is a regular language, then so is $\overline{L_1}$**
  - How to generate a DFA $\bar{M}$ that accepts $\overline{L_1}$?
    - ❖Consider the following DFA M such that $L(M) = \{w11 \mid w \in \{0,1\}^*\}$



Closure properties of regular languages

- **If L_1 is a regular language, then so is $\overline{L_1}$**
  - How to generate a DFA $\bar{M}$ that accepts $\overline{L_1}$?
    - ❖ DFA $\bar{M}$ that can accept $L(\bar{M})$ is constructed as follows



Closure properties of regular languages

- **If L_1 is a regular language, then so is $\overline{L_1}$**
  - Consider a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that accept L_1
  - We can construct DFA $\bar{M} = (Q, \Sigma, \delta, q_0, \boldsymbol{Q - F})$ that accept $\overline{L_1}$
  - Therefore, $\overline{L_1}$ is a regular language

Closure properties of regular languages

- **If L_1 and L_2 are regular languages, then so is $L_1 \cap L_2$**
  - Using DeMorgan's Law
    - $\overline{A \cup B} = \bar{A} \cap \bar{B}$
    - $\overline{A \cap B} = \bar{A} \cup \bar{B}$
  - $L_1 \cap L_2 = \overline{(\overline{L_1} \cup \overline{L_2})}$
    - If L_1 and L_2 are regular languages, then so are $\overline{L_1}$ and $\overline{L_2}$
    - If $\overline{L_1}$ and $\overline{L_2}$ are regular languages, then so are $\overline{L_1} \cup \overline{L_2}$
    - If $\overline{L_1} \cup \overline{L_2}$ is a regular language, then so is $\overline{(\overline{L_1} \cup \overline{L_2})}$
  - Therefore, $L_1 \cap L_2$ is a regular language

Closure properties of regular languages

- **If L_1 and L_2 are regular languages, then so is $L_1 - L_2$**
  - We can use the following fact
    - $L_1 - L_2 = L_1 \cap \overline{L_2}$
  - Therefore, $L_1 - L_2$ is a regular language

Next Lecture

- **Pumping Lemma: Identifying non-regular languages**